Computing with Belief Networks

Tuesday, November 5, 2019 1:58 PM

Variable Elimination "VE" Algorithm:-

- Manipulate probability tables as "factor":

factor = a representation of a function from Random Variable Values to numbers.

Example

Random Variables X, Y, Z = they are boolean

X	Y	Z	val
t	t	t	0.1
t	t	f	0.9
t	f	t	0.2
t	f	f	0.8
f	t	t	0.4
f	t	f	0.6
f	f	t	0.3
f	f	f	0.7

$r(X,Y,Z)$

A probability distribution is a factor with the extra constraint that $\sum val = 1$

- Operations on "factors"
  - fix value of a variable

$r(X=t, Y, Z) = r'(Y,Z) =$

Y	Z	
t	t	0.1
t	f	0.9
f	t	0.2
f	f	0.8

  - Multiply Factors:

$f_1(A,\underline{B}) \quad f_2(\underline{B},C)$

$f_1 * f_2 = f_3(A,B,C) = f_1(A,B) \cdot f_2(B,C)$

Example:

f_1

A	B	
t	t	0.1
t	f	0.9
f	t	0.2
f	f	0.8

$\cdot$

f_2:

B	C	
t	t	0.3
t	f	0.7
f	t	0.6
f	f	0.4

f_3:

A	B	C		
t	t	t	0.03	0.1×0.3
t	t	f	0.07	0.1×0.7
t	f	t	0.54	0.9×0.6
t	f	f	0.36	
f	t	t	0.06	0.2×0.3
f	t	f	0.14	
f	f	t	0.48	
f	f	f	0.32	

  - Sum a variable out.

$\sum_B f_1(A,B,C) = f_2(A,C)$

$$f_2(A,C) = f_1(A, B=t, C) + f_1(A, B=f, C)$$

Example:

$f_1 =$

A	B	C	val
t	t	t	0.03
t	t	f	0.07
t	f	t	0.54
t	f	f	0.36
f	t	t	0.06
f	t	f	0.14
f	f	t	0.48
f	f	f	0.32

$\sum_B f_1 = f_2$

A	C	val	
t	t	0.57	0.03 + 0.54
t	f	0.43	0.36 + 0.07
f	t	0.54	0.06 + 0.48
f	f	0.46	0.14 + 0.32.

- Posterior probability

want : $P(Z \mid Y_1 = v_1, Y_2 = v_2, Y_3 = v_3 \dots Y_n = v_n)$

$$p(h \mid e) = \frac{p(h \wedge e)}{p(e)}$$

$$= \frac{P(Z, Y_1 = v_1, Y_2 = v_2, Y_3 = v_3 \dots Y_n = v_n)}{P(Y_1 = v_1, Y_2 = v_2, Y_3 = v_3 \dots Y_n = v_n)}$$

equal.

$$= \frac{P(Z, Y_1 = v_1, Y_2 = v_2, \dots Y_n = v_n)}{\sum_Z P(Z, Y_1 = v_1, Y_2 = v_2, \dots Y_n = v_n)}$$

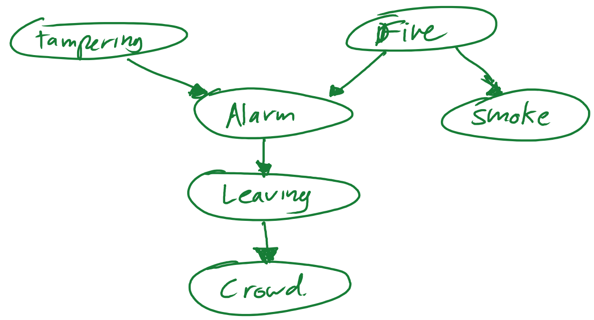
To compute $P(Z, Y_1 = v_1, Y_2 = v_2, \dots Y_n = v_n)$ the VE algorithm

1) Construct a factor from each conditional probability
2) Set the observed variables Y to their observed values.
3) Sum out all other variables. (In some ordering)
4) Multiply remaining factors and <u>Normalize</u> $\quad f(Z) / \sum_Z f(Z) =$

EXAMPLE:

Recall the alarm domain.

Recall the alarm domain.



$$P(T \mid S=t \wedge C=t) = ?$$

Conditional Probability Tables	factors
$P(T)$	$f_0(T)$
$P(F)$	$\times f_1(F)$
$P(A \mid F,T)$	$\times f_2(A,F,T)$
$P(S \mid F)$	$\times f_3(F)$ $S=$ true.
$P(L \mid A)$	$\times f_4(L,A)$
$P(C \mid L)$	$\times f_5(L)$ $C=$ true

Eliminate Fire

$$f_1(F) * f_2(A,F,T) \cdot f_3(F) = f(A,F,T)$$

$$\sum_F f(A,F,T) = \times f_6(T,A)$$

Eliminate Alarm:

$$f_4(L,A) * f_6(T,A) = f(L,A,T) \qquad \sum_A f(L,A,T) = \times f_7(L,T)$$

Eliminate Leaving:

$$f_5(L) \cdot f_7(L,T) = f(L,T) \qquad \sum_L f(L,T) = f_8(T)$$

$f_0(T) \cdot f_8(T) = f_9(T)$ is prop. $P(T, S=t \wedge C=t)$

$$\frac{f_9(T)}{\sum_T f_9(T)} = P(T \mid S=t \wedge C=t) \qquad P(h \mid e) = \frac{P(h \wedge e)}{P(e)}$$

$$\sum_T f_9(T) \rightarrow P(\underline{S=t \wedge C=t})$$